Lab 4

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BIOSTAT 100A Summer Session C 2024

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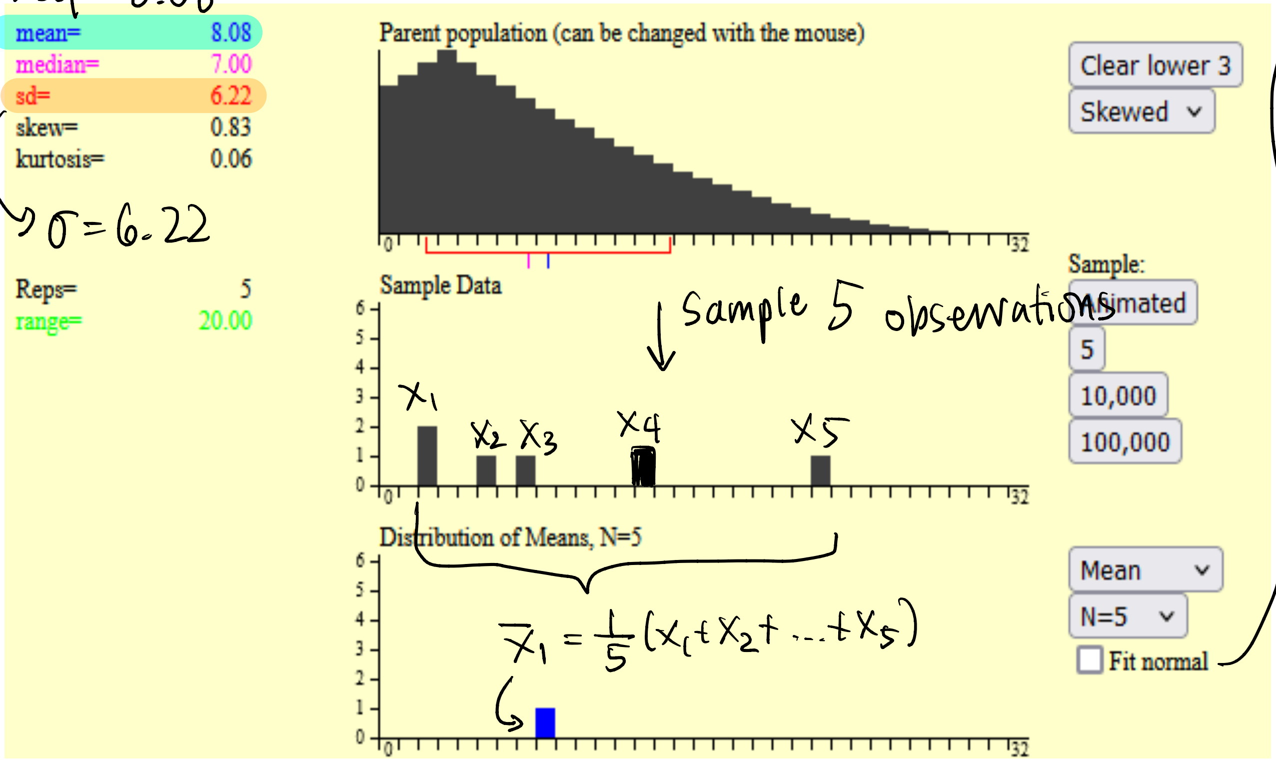
Properties of the Sampling Distribution of $\bar{x}$

What is a Sampling Distribution?

distribution of a sampling statistic

$\mu = 8.08$

$\sigma = 6.22$



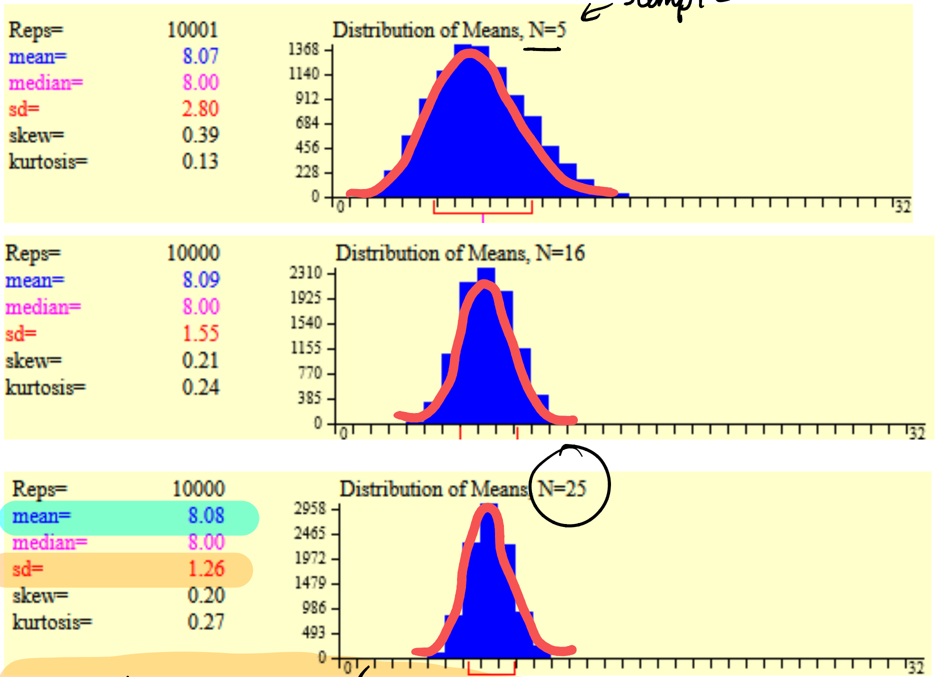
Sample 5 observations

x_1 x_2 x_3 x_4 x_5

$\bar{x}_1 = \frac{1}{5}(x_1 + x_2 + \dots + x_5)$

10,000 iterations later...

samples



$sd(\bar{x}) = 1.26 \approx \frac{6.22}{\sqrt{25}} = 1.24$

What do you notice?

- *About the Shape?* as we sample, the distribution of the means becomes normal
- *What happens as we increase the sample size?* distribution is more narrow as the sample size ↑

→ as ↑ n (sample size), ↓ variance / sd.

Properties of the Sampling Distribution of $\bar{x}$

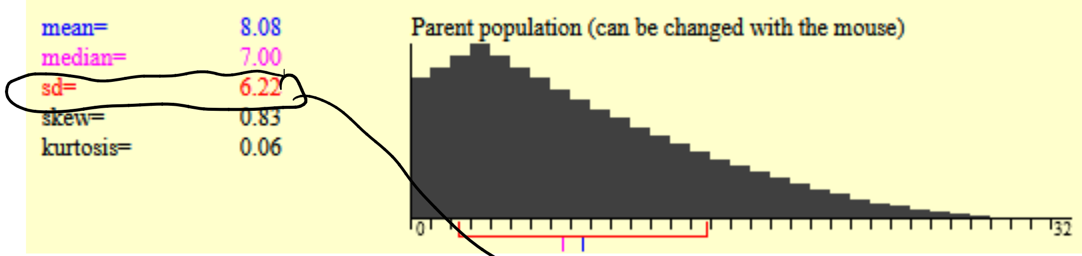
(1) $\text{Mean}(\bar{x}) = \mu$ (Population Mean) $\Rightarrow$ $\bar{x}$ is an **unbiased estimator** of μ

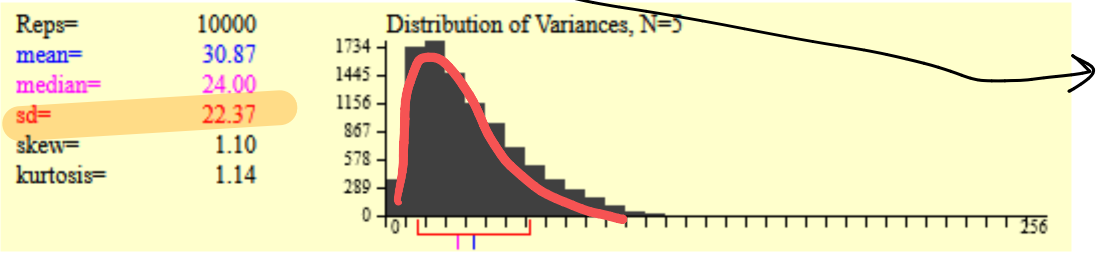
$\mathbb{E}(\bar{x}) = \mu$ ["Expectation of $\bar{x}$"]

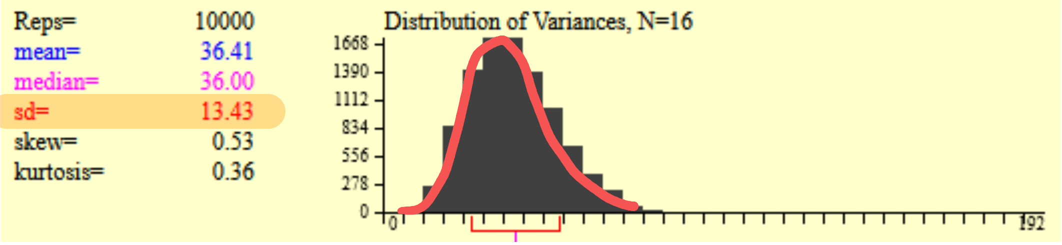
(2) $sd(\bar{x}) = \frac{\sigma}{\sqrt{n}}$ $\leftarrow$ **Standard Error** of Mean (SEM)

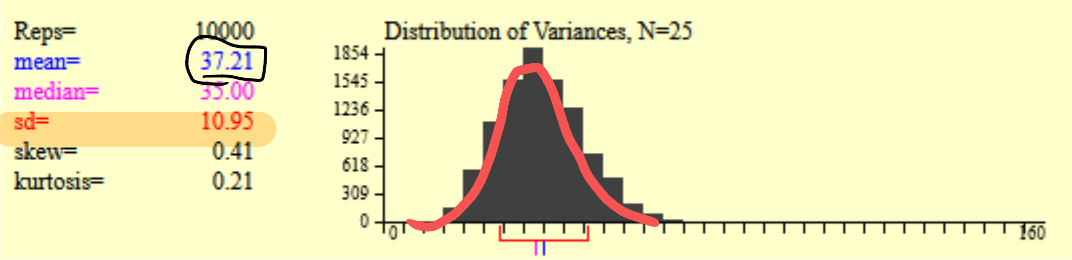
(3) Shape of Distribution is **Normal**

Let’s do the same thing for the variances…









What do you notice?

- *About the Shape?* more normal
- *What happens as we increase the sample size?*

more normal, sd ↓

$\sigma^2 = 6.22^2 \approx 36$

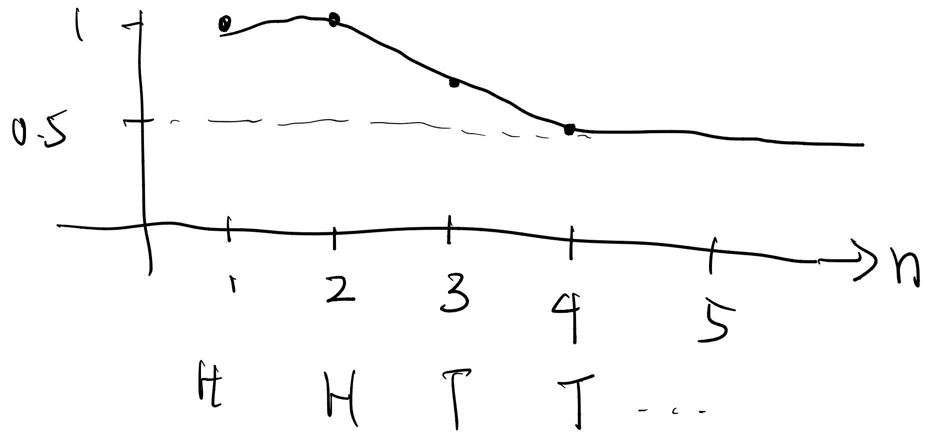
$E(s^2) = 37.21$

Properties of Sampling Distribution of s^2, $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ (sample variance)

$\text{Mean}(s^2) = \mathbb{E}(s^2) = \sigma^2 \Rightarrow s^2$ is an **unbiased estimator** of σ^2

Law of Large Numbers - if we perform a statistical many, many times then probabilities of an event converge

Ex: $\Pr(\text{heads}) = \frac{1}{2}$



Competency Assessment

- In a family of 3 children, what is the probability of 2 boys and 1 girl?

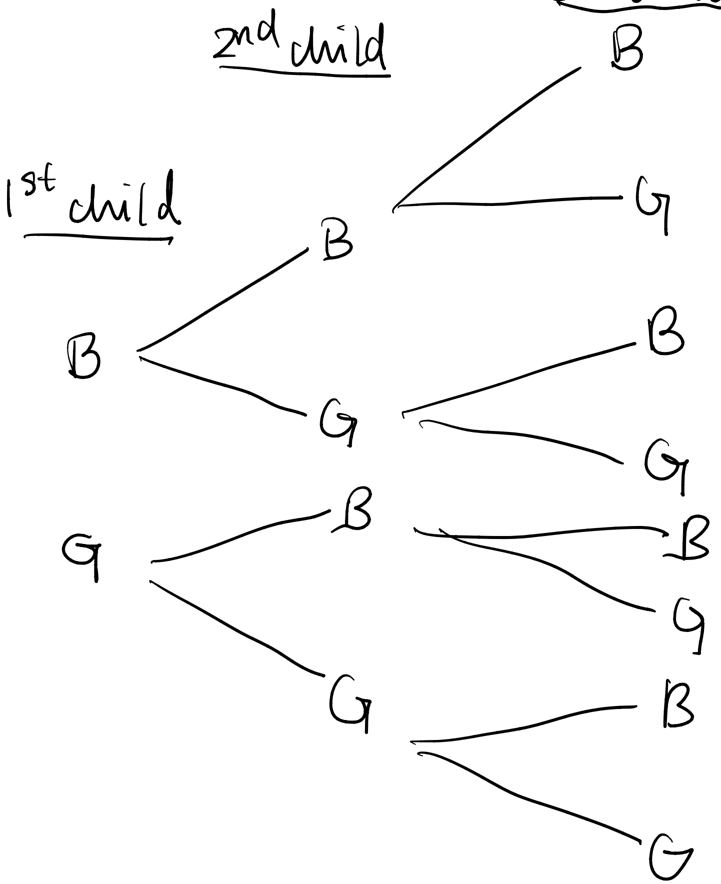
_ _ _

B B B

B B G

B G B

⋮



Possibilities

1. BBB
2. BBG
3. BGB
4. BGG
5. GBB
6. GBG
7. GGB
8. GGG

$$\boxed{\frac{3}{8}}$$

3 ← # of possibilities that meet criteria

8 ← Total # of possibilities.

Competency Assessment

"success" as having a girl.

$3! = 3 \cdot \cancel{2 \cdot 1}$
$2! = \cancel{2 \cdot 1}$

- In a family of 3 children, what is the probability of 2 boys and 1 girl?

Using Binomial Distribution, $n=3$, $k=1$, $p=1/2$

$$Pr(k=1) = \underbrace{\binom{3}{1}}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{3-1} = \frac{\cancel{3!}\,3}{1!\,\cancel{2!}}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = 3\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)^2 = \frac{3}{2^3} = \boxed{\frac{3}{8}}$$

# of ways you can pick 1 girl out of 3 kids;
# of combinations to give the desired result.

Binomial Distribution

1. Fixed n (sample size)
2. Only two possible outcomes: "success" or "failure"
3. Probability of "success", p, is constant
4. Trials are independent.

Let n = sample size, k = # of "successes", p = probability of success = Pr("success")

$$P(K=k) = \underbrace{\binom{n}{k}}_{\text{"n choose k"}} p^k (1-p)^{n-k} \quad \text{where } \binom{n}{k} = \frac{n!}{k!(n-k)!}$$