Lab 6

Confidence Intervals and T-Tests

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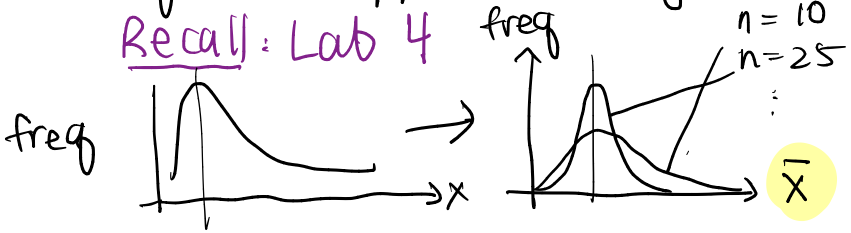
Normal Distribution and CLT

- Central Limit Theorem: Sampling distribution of $\bar{x}$ is approximately normal if n is sufficiently large

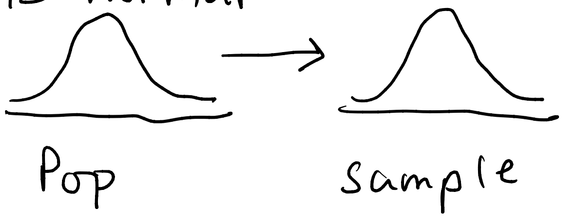
  $n \to \infty \Rightarrow \bar{x} \sim \text{Normal}$

  Recall: Lab 4



- When is a sample "sufficiently large"? $n \geq 30$

- When do we assume normality? (1) $n \geq 30$ by CLT (large sample)
  (2) if n is small ($n < 30$), assume we are sampling from a population that is normal



Lecture 1/2

- Inference: (1) Estimation vs. (2) Hypothesis Testing

  (1) placing reasonable value(s) on a chosen pop'n parameter (i.e. Confidence Intervals)

  (2) Hypothesize values for a pop'n parameter and infer whether its reasonable or not.

→ Z-Test

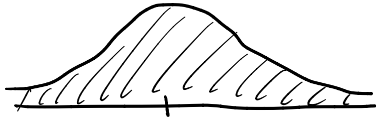
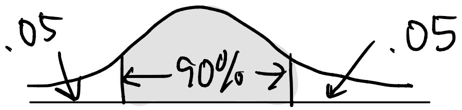
Lecture 8: Confidence Intervals under known σ^2 (1)

population variance

- Interpretation of a Confidence Interval
  - Math: $P(\text{lower bound} < \mu < \text{upper bound}) =$ Confidence Level (i.e. 0.95)
  - In words: The probability that our sampled $\bar{x}$ falls between (LB, UB) is [Confidence Level]

Lab: If we created 100 95% Confidence Intervals for μ, 95 of them would contain μ (pop'n mean)

- Common Confidence Intervals for μ:

Confidence Level	α	z	Confidence Interval $\bar{x} \pm \underbrace{z_{1-\alpha/2}\, \sigma/\sqrt{n}}_{\text{Margin of Error (MOE)}}$	Picture Interpretation	Margin of Error
100% CI	0		$-\infty < \mu < +\infty$		$+\infty$
99 % CI 1-.99	0.01	2.58	$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$		$2.58 \frac{\sigma}{\sqrt{n}}$
95% CI 1-.95	0.05	1.96	$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$		$1.96 \frac{\sigma}{\sqrt{n}}$
90% CI 1-.9	0.10	1.64	$\bar{x} \pm 1.64 \frac{\sigma}{\sqrt{n}}$		$1.64 \frac{\sigma}{\sqrt{n}}$

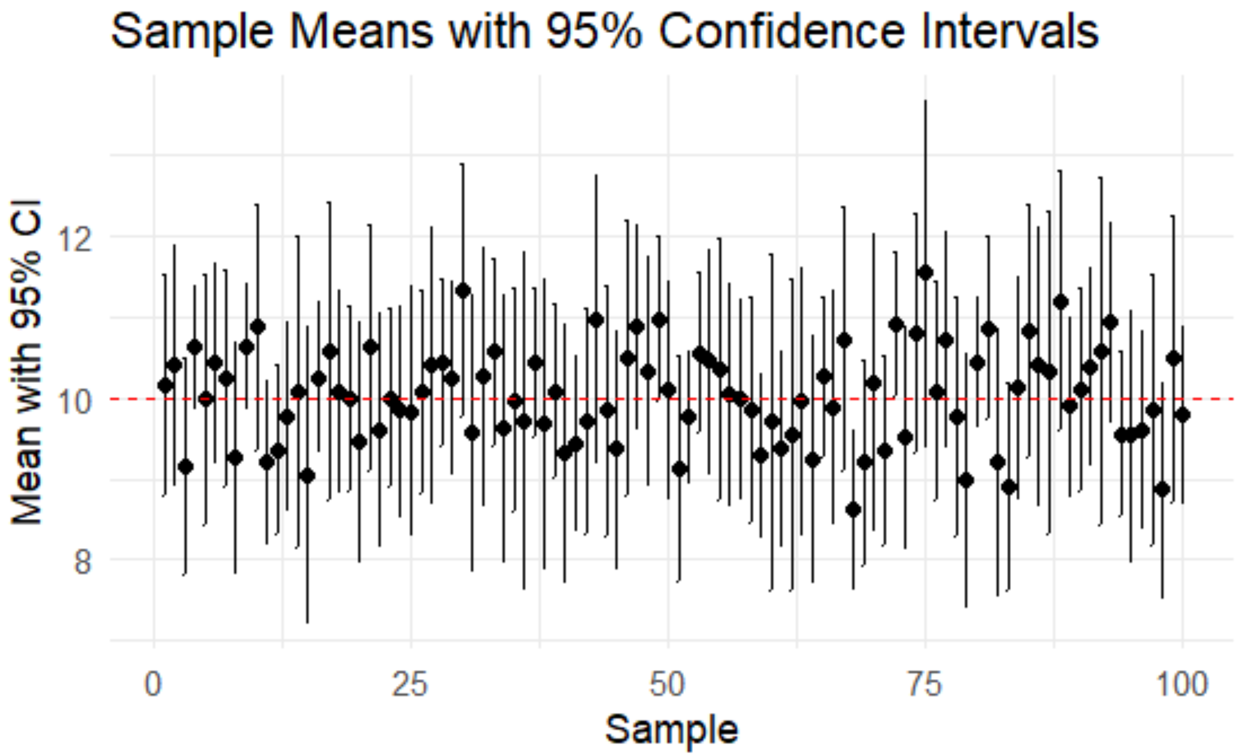
$\mu=10,\ \sigma^2=4$

sample size: $n=10$

Samples from N(10,4)					
Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
8.879049	12.448164	7.864353	10.852928	8.610586	10.506637
9.539645	10.719628	9.564050	9.409857	9.584165	9.942906
13.117417	10.801543	7.947991	11.790251	7.469207	9.914259
10.141017	10.221365	8.542217	11.756267	14.337912	12.737205
10.258575	8.888318	8.749921	11.643162	12.415924	9.548458
13.430130	13.573826	6.626613	11.377280	7.753783	13.032941
10.921832	10.995701	11.675574	11.107835	9.194230	6.902494
7.469877	6.066766	10.306746	9.876177	9.066689	11.169227
8.626294	11.402712	7.723726	9.388075	11.559930	10.247708
9.108676	9.054417	12.507630	9.239058	9.833262	10.431883

$$CI: \bar{x}_i \pm 1.96\frac{\sigma}{\sqrt{n}} \Rightarrow \boxed{\bar{x}_i \pm 1.96\frac{2}{\sqrt{10}}}$$

Sample Means with 95% Confidence Intervals				
Sample	Mean	Lower CI	Upper CI	Include Mean?
1	10.15	8.78	11.51	1
2	10.42	8.93	11.90	1
3	9.15	7.82	10.48	1
4	10.64	9.89	11.40	1
5	9.98	8.43	11.53	1
6	10.44	9.22	11.67	1

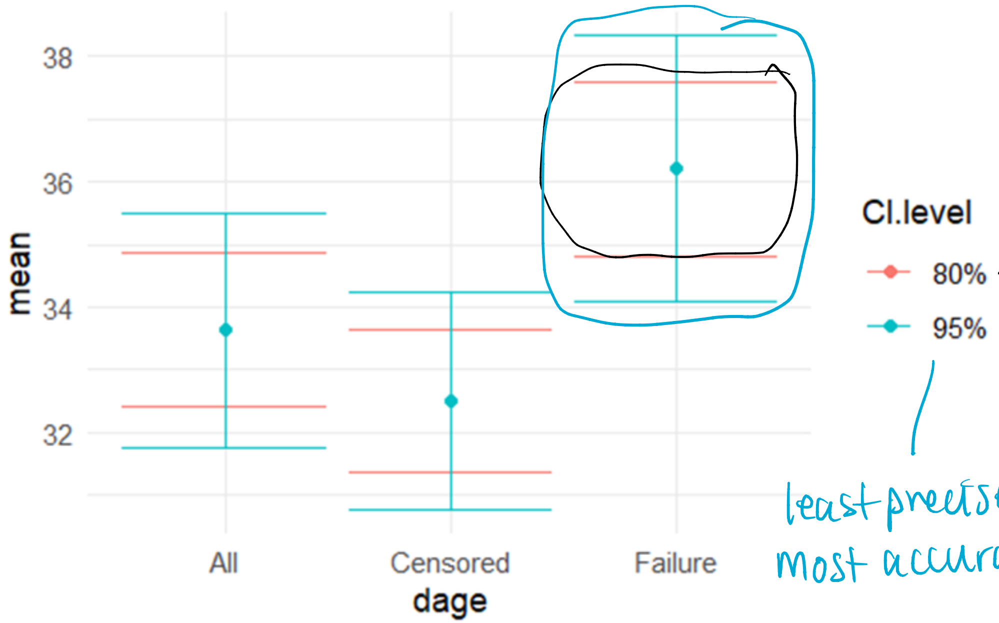


95 of these CIs will include 10

$$\frac{\text{sum(include mean)}}{\#\text{ of samples}} = \frac{1+1+0+1+\dots+0}{100 \text{ samples}} = \frac{95}{100} = 95\%$$

Lecture 8: Confidence Intervals under **known σ^2** (2)

- Accuracy vs. Precision
  - Accuracy = Confidence Level
  - Precision = Margin of Error (MOE)
  - What is the relationship between accuracy (Confidence Level) and precision (MOE)? → Inverse Relationship s.t. ↑ Accuracy, ↓ Precision & v.v.



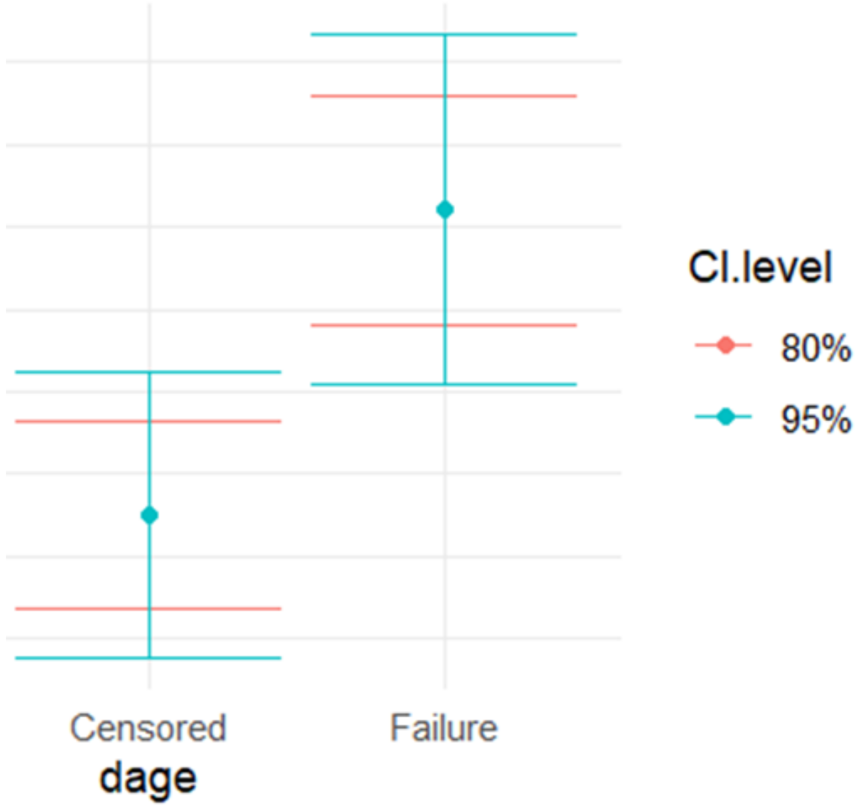
least precise, most accurate

most precise, lowest accuracy

Highest Accuracy → 99% CI → $MOE = 2.58\frac{\sigma}{\sqrt{n}}$ ← Largest MOE ⇓ Least Precise

95% CI → $MOE = 1.96\frac{\sigma}{\sqrt{n}}$

Lowest Accuracy → 90% CI → $MOE = 1.64\frac{\sigma}{\sqrt{n}}$ ← Smallest MOE ⇓ Most Precise



What are the Assumptions to create these Confidence Intervals?

(1) Simple Random Sample

(2) Know $\sigma^2 \Rightarrow z^*$ cutoffs

(3) Data is Normal

Interpret your Confidence Interval for 95%

If we create 100 CIs for dage, 95 of them will capture the true mean.

Probability of our sampled mean falls between LB and UB is 95%.

→ T-Tests

Confidence Intervals when σ^2 unknown

- Why use a t-distribution?

  We cannot replace σ with s bc our distribution will no longer be normal!

- T-distribution vs. z-distribution
  - t-value > z-value, always.
  - As degrees of freedom (df) ↑ ; t-distribution approaches z-distr.

    why? $df = n-1$

    (NOT a proof) ↑ df ⇒ n ↑ ⇒ converge to Normal distr.

    by CLT

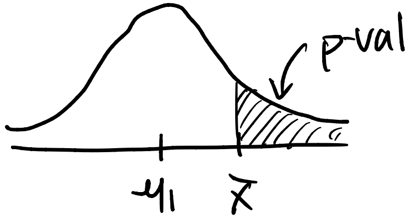
Confidence Intervals Summary

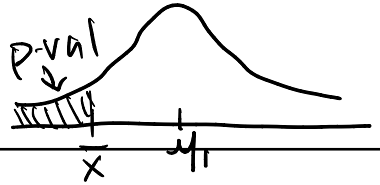
	σ^2 known	σ^2 unknown
Distribution	Normal Distribution (z)	T-distribution (t)
Margin of Error (MOE)	$z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}$	$t^*_{df=n-1}\frac{s}{\sqrt{n}}$
Confidence Interval (CI)	$\bar{x} \pm z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm t^*_{df=n-1}\frac{s}{\sqrt{n}}$
Find sample size based on MOE (n= ?) Use Algebra to Back-solve.	$MOE = z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}$ $\sqrt{n} = \frac{z_{1-\alpha/2}\,\sigma}{MOE} \Rightarrow \boxed{n = \left(\frac{z_{1-\alpha/2}\,\sigma}{MOE}\right)^2}$	$MOE = t^*_{df=n-1}\frac{s}{\sqrt{n}}$ $\boxed{n = \left(\frac{t^* s}{MOE}\right)^2}$

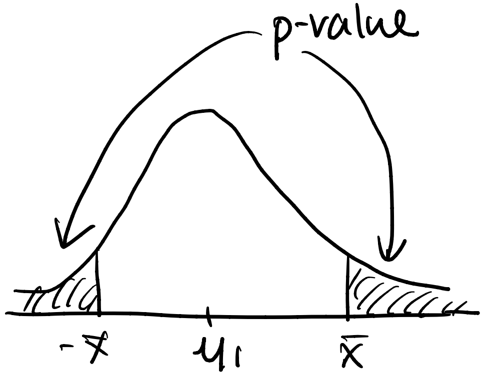
Hypothesis Testing for μ

- Interpretation of the p-value: The probability of achieving the sample result or something more extreme if H_0 is true

- Hypothesis Testing Steps on Next Slide – practice these problems!

<table>
<tr><th>Step</th><th>σ^2 known</th><th>σ^2 unknown</th></tr>
<tr><td>(1) Hypothesis</td><td colspan="2">$\boxed{\text{1-Tailed Test}}$
① $H_0: \mu = \mu_1$
$H_A: \mu > \mu_1$


② $H_0: \mu = \mu_1$
$H_A: \mu < \mu_1$


$\boxed{\text{2 Tailed Test}}$
③ $H_0: \mu = \mu_1$
$H_A: \mu \neq \mu_1$

</td></tr>
<tr><td>(2) Data</td><td>$\bar{x}, \sigma, n, \alpha$</td><td>$\bar{x}, s, n, \alpha$</td></tr>
<tr><td>(3) Statistical Test</td><td>1 Tailed Z-Val: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
2 Tailed z-val: Find the p-val. for 1-Tailed & multiply by 2</td><td>1-Tailed T-value: $t_{df=n-1} = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$</td></tr>
<tr><td>(4) Assumptions</td><td colspan="2">(1) Simple Random Sample (SRS)
(2) Normality
either by $n \geq 30 \Rightarrow$ CLT
$n < 30 \Rightarrow$ sampling from normal pop'n.</td></tr>
</table>

Step	σ^2 known	σ^2 unknown
(5) Decision Rule DO NOT EVER say "we accept the null hypothesis"	$p < 0.05$ $p > 0.05$ reject. α fail to reject $(\alpha = 0.05)$	$t < t^*_{crit.}$ $t > t^*_{crit}$ reject ↑ fail to rej. t^*_{crit}
(6) Calculation	See above	See above
(7) Statistical Decision	Reject Ho b/c p-val < α OR Fail to Reject Ho b/c p-val > α	Reject Ho b/c t-value < t^*_{crit} OR Fail to Reject Ho b/c t-val > t^*_{crit}.

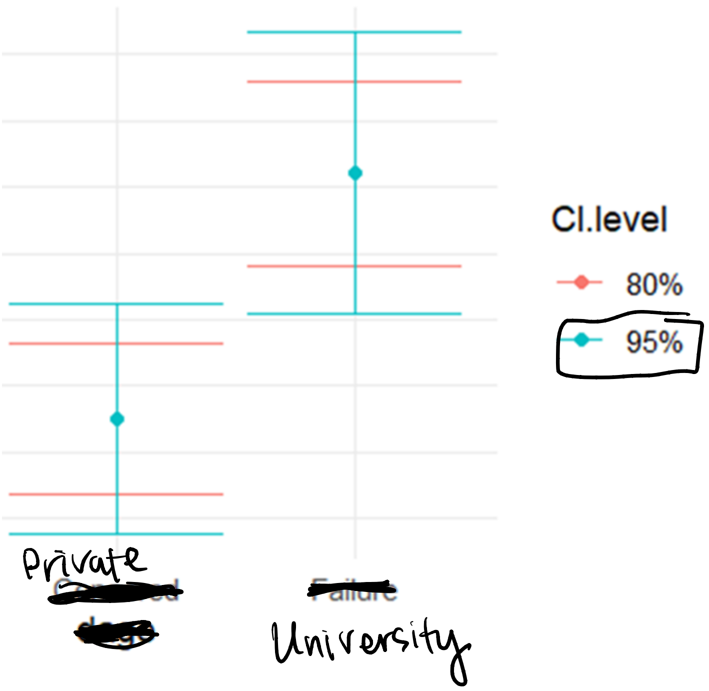
(8) Practical Decision	if reject Ho:
	"There is evidence to suggest that [variable] is [greater than / less than / other than] [μ_1]"
	(gt) (lt) (ot)
	Never abbreviate these on an exam pls.
	if fail to reject Ho:
	"There is insufficient evidence to suggest that [variable] is [gt / lt / ot] [μ_1]"	

Confidence Interval Method of Hypothesis Testing:

Test Ho: $\mu_{private}$ = $\mu_{university}$ versus Ha: not so, at α=0.05

$H_A: \mu_{private} \neq \mu_{university}$

How would we do this? *Hint: what would have to change in the graph below?*



overlapping intervals ⇒ fail to reject Ho

separate intervals ⇒ reject Ho