

One-Sample Inference (Lectures 5 & 6)

BIOSTAT 201A Fall 2025

Discussion 3 – October 17, 2025

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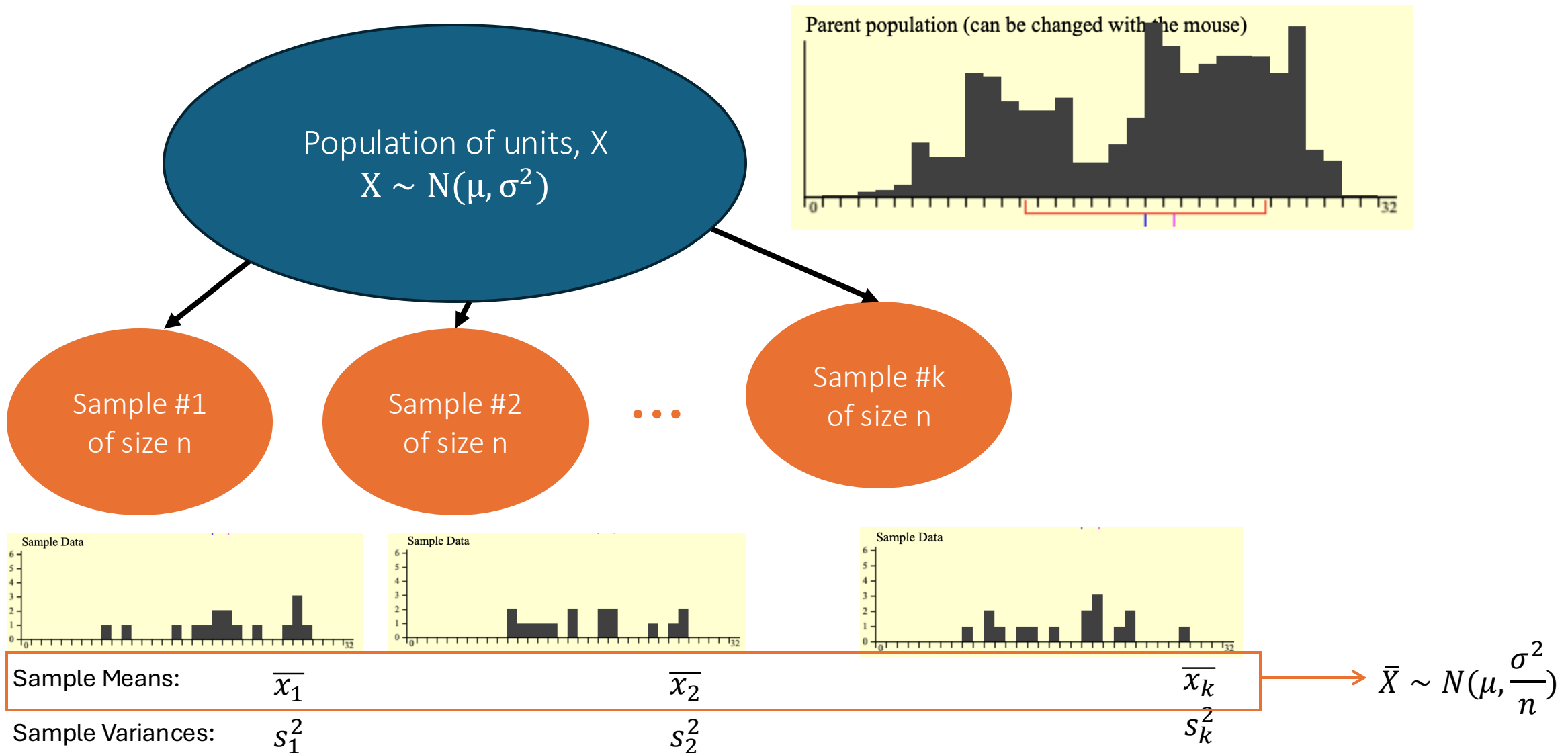
Outline

1. Lecture 4. Point and Confidence Interval Estimation for Population Mean
 1. Sampling Distribution of the Mean
 2. Confidence Intervals for Means
2. Lecture 5. Point and Confidence Interval Estimation for Population Proportion
 1. Binomial Distribution
 2. Normal Approximation to the Binomial Distribution

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1.1 Sampling Distribution of the Mean



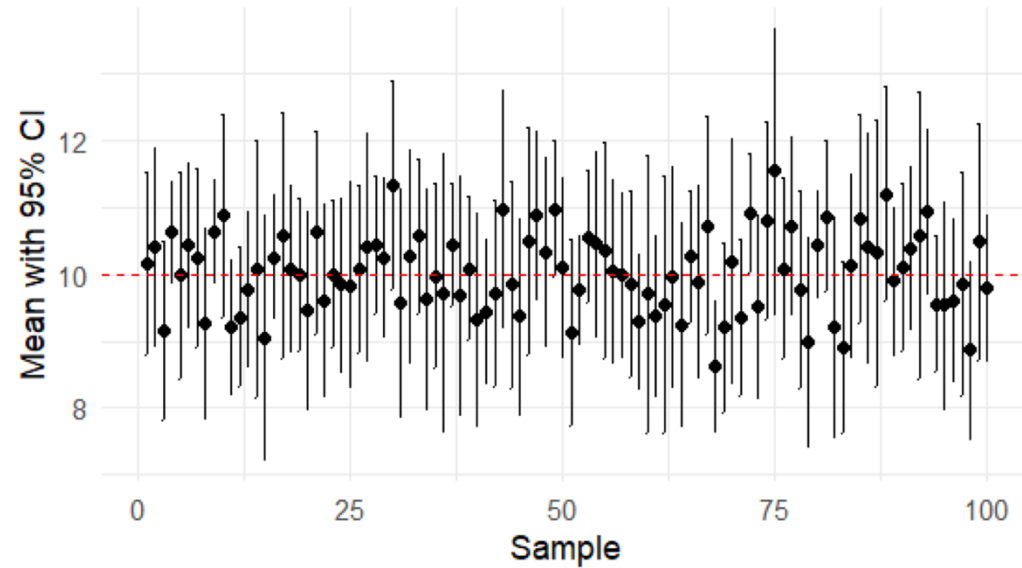
1.2 Confidence Intervals for Mean (Intuition)

Samples from N(10,4)					
Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
8.879049	12.448164	7.864353	10.852928	8.610586	10.506637
9.539645	10.719628	9.564050	9.409857	9.584165	9.942906
13.117417	10.801543	7.947991	11.790251	7.469207	9.914259
10.141017	10.221365	8.542217	11.756267	14.337912	12.737205
10.258575	8.888318	8.749921	11.643162	12.415924	9.548458
13.430130	13.573826	6.626613	11.377280	7.753783	13.032941
10.921832	10.995701	11.675574	11.107835	9.194230	6.902494
7.469877	6.066766	10.306746	9.876177	9.066689	11.169227
8.626294	11.402712	7.723726	9.388075	11.559930	10.247708
9.108676	9.054417	12.507630	9.239058	9.833262	10.431883



Sample Means with 95% Confidence Intervals				
Sample	Mean	Lower CI	Upper CI	Include Mean?
1	10.15	8.78	11.51	1
2	10.42	8.93	11.90	1
3	9.15	7.82	10.48	1
4	10.64	9.89	11.40	1
5	9.98	8.43	11.53	1
6	10.44	9.22	11.67	1

Sample Means with 95% Confidence Intervals



1.2 Confidence Intervals for Mean

Scenario	Assumptions	Confidence Interval for Mean
Known σ^2 , sample size $n > 30$	No Assumptions due to Central Limit Theorem!	$\bar{x} \pm z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}$
Unknown σ^2 and large n ($n > 200$)	Population is (approximately) normal	$\bar{x} \pm z_{1-\alpha/2} \frac{s}{\sqrt{n}}$
Unknown σ^2 and small n ($n \leq 200$)		$\bar{x} \pm t_{n-1, 1-\alpha/2} \frac{s}{\sqrt{n}}$

Exercise.

Pulmonary Disease

The data in Table 6.10 concern the mean triceps skin-fold thickness in a group of normal men and a group of men with chronic airflow limitation [5].

TABLE 6.10 Triceps skin-fold thickness in normal men and men with chronic airflow limitation

Group	Mean	sd	n
Normal	1.35	0.5	40
Chronic airflow limitation	0.92	0.4	32

Source: Adapted from *Chest*, 85(6), 58S–59S, 1984.

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- a) Compute the Standard Error of the Mean for the Normal Group
- b) What is an Unbiased Estimator of the Population Mean?
- c) What is an Unbiased Estimator of the Population Variance?
- d) Compute a 95% Confidence Interval for the Mean of the Normal Group
- e) Compute a 90% Confidence Interval for the Mean of the Normal Group
- f) Compare the 90% vs. 95% Confidence Intervals. Which one is larger, and what does that tell you about the precision of the estimate?

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2.1 The Binomial Distribution

A **binomial random variable** counts the number of “successes” in the n independent trials, where each trial has a probability p of success

$$X \sim \text{Binomial}(n, p)$$

where $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$ where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ or “n-choose-k”

Conditions:

1. Experiment consists of n identical and independent (iid) trials
2. Dichotomous outcomes (only two possible outcomes) on each trial
3. $\Pr(\text{“success”}) = p$, $\Pr(\text{“failure”}) = 1-p$ where p is constant
4. Variable of interest is the number of successes observed during the n trials

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Exercise. Let $p = 0.2$ and $n = 5$.

(a) What is the Expected Number of Successes?

(b) What is the Variance of Successes?

(a) Compute the $\Pr(X \leq 3)$.

2.2 Normal Approximation to the Binomial Distribution

What you want to Estimate	Distribution	Assumption	Confidence Interval
Number of Successes	$X \sim N(np, np(1 - p))$	$np(1 - p) \geq 5$	
Proportions	$\hat{p} \sim N(p, \frac{p(1-p)}{n})$ where $\hat{p} = \frac{X}{n}$		$\hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$

Exercise. You obtain a simple random sample of 80 individuals, and 52 report having received a flu shot.

- What is $\hat{p}$?
 - What is the standard error of $\hat{p}$?
 - What is the 95% confidence interval for p ?
- d) Now suppose in a smaller nearby community, where you collect data from 80 individuals, and 12 have received a flu shot. Can you use the normal approximation to compute a 95% confidence interval? If not, explain why.