Two-Sample Inference (Lectures 8-10)

BIOSTAT 201A Fall 2025

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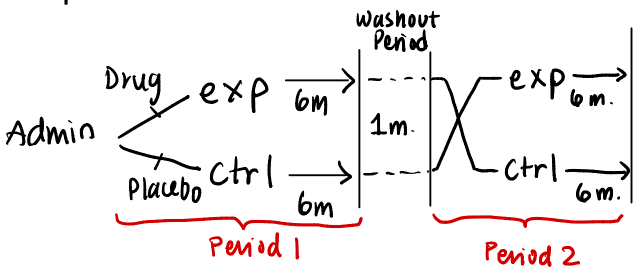
Outline

1. Motivation for Two Sample Inference Problems
2. Two Sample Inference for Paired Data (Lecture 8)
3. Two Sample Inference for Independent Samples (Lecture 9 & 10)

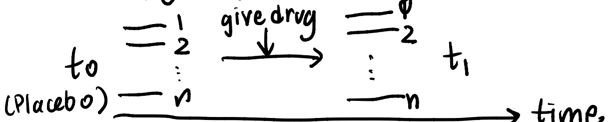
Two Sample Problems

- Goal of Two Sample Problems:
  - (1) Compare 2 Samples
  - (2) Identify Important Covariates (i.e. Males vs. Females, Young vs. Old, etc)
- Types
  - 1. Stratified Randomization (Independent Samples)
  - 2. Matched-Pairing (Dependent Samples)
    - Unpaired is more likely to reject but why should we still use a paired test?

      if there is a reason to do do matched pairs; we SHOULD do it, bc it helps w/ controlling for covariates (Goal #2)
    - Situations where we do matched-pairs in clinical research:
      - 1. Cross-Over Design



      - 2. Pre-Post Design



Two-Sample Problems	Dependent Groups (Paired Data)
Hypotheses	Let $\Delta = \mu_1 - \mu_2$ $H_0: \Delta = 0$ vs. $H_1: \Delta \neq 0$
Data	$\begin{array}{c\|cc\|l} i & x_{1i} & x_{2i} & d_i \\ \hline 1 & x_{11} & x_{21} & d_1 = x_{11} - x_{21} \\ 2 & x_{12} & x_{22} & d_2 = x_{12} - x_{22} \\ \vdots & \vdots & \vdots & \vdots \\ n & x_{1n} & x_{2n} & d_n = x_{1n} - x_{2n} \end{array}$ where $d_i \sim N(\Delta, \sigma_d^2)$
Assumptions	The population of differences is normally distributed with mean Δ and variance σ_d^2
Test Statistic	$t = \dfrac{\bar{d}}{S_d/\sqrt{n}}$
Decision Rule	$\|t\| > t_{df=n-1,\, 1-\alpha/2} \Rightarrow$ Reject H_0 $\|t\| < t_{df=n-1,\, 1-\alpha/2} \Rightarrow$ Fail to reject H_0
Confidence Interval	$\bar{d} \pm t_{n-1,\, 1-\alpha/2} \dfrac{S_d}{\sqrt{n}}$ where $SE(d) = \dfrac{S_d}{\sqrt{n}}$

	Raw scale urinary protein (g/24 hr)		Diff
Patient	Before	After	
1	25.6	10.1	15.5
2	17.0	5.7	11.3
3	16.0	5.6	⋮
4	10.4	3.4	
5	8.2	6.5	
6	7.9	0.7	
7	5.8	6.1	
8	5.4	4.7	
9	5.1	2.0	
10	4.7	2.9	1.8

$\bar{d} = 5.84$

$s_d = 5.29$

Renal Disease. We are interested to see if the raw scale of urinary protein changes after 8 weeks.

(a) Identify the appropriate statistical procedure to do this. Explain.

Paired T-test since data are paired; dependence

(b) Did urinary protein change after 8 weeks? Use a hypothesis test to determine this.

$H_0: \Delta = 0$ vs. $\Delta \neq 0$

"no change" "changed"

Assuming the population of differences is normally distributed with mean Δ and variance σ_d^2,

$$TS = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{5.84}{5.29/\sqrt{10}} = 3.49, \quad t_{df=9,0.975} = 2.26$$

Since $TS = 3.49 > 2.26$, we $\underline{\text{reject}}$ the null hypothesis and conclude that there is a change in urinary protein after 8wks.

(c) Compute a 95% confidence interval for the differences in urinary protein change. How does your created confidence interval support your conclusions in part (b)?

$$\bar{d} \pm t_{9,0.975}\frac{s_d}{\sqrt{n}} = 5.84 \pm 2.62\left(\frac{5.29}{\sqrt{10}}\right) = (2.06, 9.62)$$

Since our CI does not include 0, we conclude that there is a significant difference in urinary protein change like we did in part (b)

Two-Sample Problems	Independent Groups		
	Equal Variances	Unequal Variances	Test for Equality of Variances
Hypotheses	$H_0: \mu_1 = \mu_2 \Leftrightarrow \mu_1 - \mu_2 = 0$ $H_1: \mu_1 \neq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \neq 0$		$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$
Data	Sample 1: $\bar{X}_1, S_1^2, n_1$ Sample 2: $\bar{X}_2, S_2^2, n_2$		
Assumptions	$X_{1i} \sim N(\mu_1, \sigma^2), X_{2i} \sim N(\mu_2, \sigma^2)$ where $\sigma^2 = \sigma_1^2 = \sigma_2^2$	$X_{1i} \sim N(\mu_1, \sigma_1^2)$ $X_{2i} \sim N(\mu_2, \sigma_2^2)$	
Test Statistic	$t = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, where $S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$	$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$	$F = \frac{S_2^2}{S_1^2}$ where $S_2 \geq S_1$
Decision Rule	$\lvert t\rvert \geq t_{df=n_1+n_2-2, 1-\alpha/2} \Rightarrow$ Reject H_0 $\lvert t\rvert < t_{df=n_1+n_2-2, 1-\alpha/2} \Rightarrow$ Fail to Reject H_0	Use Satterwaithe's Approx. for df s.t. $d' = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1-1} + \frac{(S_2^2/n_2)^2}{n_2-1}}$ and round down to nearest integer, d'' Then $\lvert t\rvert \geq t_{d'', 1-\alpha/2} \Rightarrow$ Reject H_0 $\lvert t\rvert < t_{d'', 1-\alpha/2} \Rightarrow$ Fail to Reject H_0	$F > F_{n_2-1, n_1-1, 1-\alpha/2}$ or $F < F_{n_2-1, n_1-1, \alpha/2}$ $\Rightarrow$ Reject H_0 o.w. $\Rightarrow$ Fail to Reject H_0
Confidence Interval	$(\bar{X}_1 - \bar{X}_2) \pm t_{n_1+n_2-2, 1-\alpha/2} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	$(\bar{X}_1 - \bar{X}_2) \pm t_{d'', 1-\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$	

Nutrition. The mean ±1 sd of ln [calcium intake (mg)] among 25 females, 12 to 14 years of age, below the poverty level is 6.56 ± 0.64. Similarly, the mean ± 1 sd of ln [calcium intake (mg)] among 40 females, 12 to 14 years of age, above the poverty level is 6.80 ± 0.76. Let X_{1i} = ln of females below poverty, X_{2i} = ln of females above poverty.

where $\bar{X}_1 = 6.56$, $S_1 = 0.64$, $n_1 = 25$

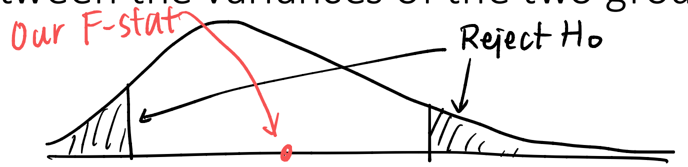
$\bar{X}_2 = 6.80$, $S_2 = 0.76$, $n_2 = 40$

$\alpha = 0.05$

(a) Test for a significant difference between the variances of the two groups.

$H_0: \sigma_1^2 = \sigma_2^2$ vs. $H_1: \sigma_1^2 \neq \sigma_2^2$

$F = S_2^2 / S_1^2 = 1.41$



$F_{39,24,0.025} = 0.49$ $\quad F_{39,24,0.975} = 2.15$

Since our test statistic is between $F_{39,24,0.025}$ and $F_{39,24,0.975}$ we fail to reject Ho and conclude that the variances of the two groups are not significantly different.

(b) Test for whether there is a significant difference in means between the two groups.

$H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$ w/ pooled variance (equality of variances s.t. $\sigma^2 = \sigma_1^2 = \sigma_2^2$)

$$TS = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ where } S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(25-1)(0.64)^2 + (40-1)(0.76)^2}{25+40-2}} = 0.717, \; t_{crit} = t_{63,0.975} = 1.99$$

$$= \frac{6.56 - 6.80}{0.717\sqrt{\frac{1}{25} + \frac{1}{40}}}$$

$$= -1.314$$

Since $|TS| < 1.99$, we fail to reject Ho and conclude that there is no significant difference in mean calcium intake between females age 12-14 above and below the poverty line.

(c) Create a 95% Confidence Interval for the difference in means between the two groups.

$(\bar{X}_1 - \bar{X}_2) \pm t_{crit} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (-0.605, 0.125)$ → includes 0 → no significant diff b/t groups.