Exercise 1. Retinitis pigmentosa is a disease that can follow different genetic modes of inheritance, including sex-linked and autosomal (non–sex-linked) forms. Researchers are interested in whether the **proportion of sex-linked cases** differs between two ethnic populations.

A survey identified retinitis pigmentosa cases in an English and a Swiss population. For this analysis, autosomal dominant and autosomal recessive forms are combined into a single **"non–sex-linked"** category. The data are:

- Among **125 English** cases, **46** were sex-linked and **79** were non–sex-linked.
- Among **110 Swiss** cases, **2** were sex-linked and **108** were non–sex-linked.

Use Fisher's Exact Test to determine if the proportion of sex-linked cases differs between two ethnic populations. (Adapted from Rosner Exercise 10.5)

Step 1. What hypotheses are you testing?

Null Hypothesis: Sex-Linked Cases and Ethnic population are independent

Alternative Hypothesis: Sex-Linked Cases and Ethnic population are dependent

Step 2. Construct the Contingency Table.

	Sex-Linked	Non-Sex Linked	
Swiss	2	108	110
English	46	79	125
	48	187	235

Step 3: Compute the probability of seeing your current table?

Let $X=2$, $K=48$, $N-K=187$, $N=235$, $n=110$. Then,

$$Pr(X=2) = \frac{\binom{48}{2}\binom{187}{108}}{\binom{235}{110}} = \boxed{7.660564e-13}$$

Step 4: Construct the other more "extreme" tables and their corresponding probabilities

1	109	110
47	78	125
48	187	235

$$Pr(X=1) = \frac{\binom{48}{1}\binom{187}{109}}{\binom{235}{110}} =$$

0	110	110
48	77	125
48	187	

$$Pr(X=0) = \frac{\binom{48}{0}\binom{187}{110}}{\binom{235}{110}} = 3.490231e-16$$

in R, To get the $p(x) = \frac{\binom{m}{x}\binom{n}{k-x}}{\binom{m+n}{k}}$ $\Rightarrow$ $m = 48$, $n = 187$, $k = 110$, $x = 1$

$\Rightarrow$ dhyper(x, m, n, k)

= dhyper(1, 48, 187, 110) = 2.362618e-14

Step 5: Add the probabilities to get your p-value.

$Pr(X \leq 2) = Pr(X=0) + Pr(X=1) + Pr(X=2)$

$= 7.660564e{-13} + 2.362618e{-14} + 3.490231e{-16}$

$\boxed{\text{p-value} = 7.9003e^{-13}}$

can also do phyper(2, 48, 187, 110) in R $= Pr(X \leq 2)$

Step 6: State your conclusion.

Since our p-value < 0.05, we conclude that Sex-Linked Cases and Ethnic populations are independent at $\alpha = 0.05$ level.

Exercise 2. Two drugs (A, B) are compared for the medical treatment of duodenal ulcer. For this purpose, patients are carefully matched with regard to age, gender, and clinical condition.

The treatment results based on 200 matched pairs show that for 89 matched pairs both treatments are effective; for 90 matched pairs both treatments are ineffective; for 5 matched pairs drug A is effective, whereas drug B is ineffective; and for 16 matched pairs drug B is effective, whereas drug A is ineffective. (Adapted from Rosner Exercises 10.8-10.12)

(a) What test procedure can be used to assess the results?

McNemar's Test

(b) Perform the test in (a) and report a p-value.

Step 1. What hypotheses are you testing?

Null Hypothesis: Drug A and Drug B are equally effective

Alternative Hypothesis: Drug A and Drug B are not equally effective.

Step 2. Construct the Matched Pairs Table.

		Treat. B. effective	Treat. B. not effective	
Treat A	effective	89	5	
	not-effective	16	90	
				200

Step 3. What are n_A, n_B, and n_D?

$$\boxed{n_A = 5,\ n_B = 16,\ n_D = n_A + n_B = 21}$$

Since $n_D \geq 20$ we can use the normal approximation.

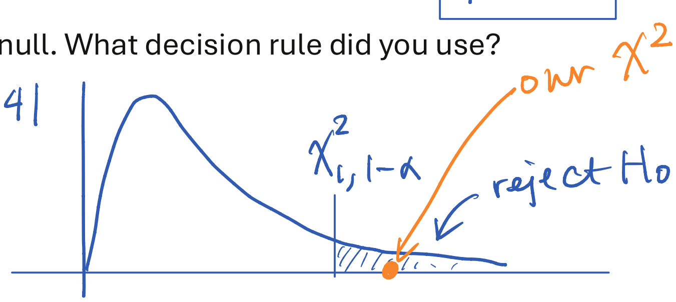
Step 4. Calculate the χ^2 test statistic.

$$\chi^2 = \left(\left|n_A - \frac{n_D}{2}\right| - \frac{1}{2}\right)^2 \Big/ \left(\frac{n_D}{4}\right) = \left(\left|5 - \frac{16}{2}\right| - \frac{1}{2}\right)^2 \Big/ \left(\frac{16}{4}\right)$$

$$= (|n_A - n_B| - 1)^2/(n_A + n_B) = (|5-16|-1)^2/(5+16) = \boxed{4.762}$$

Step 5. Determine if we reject the null. What decision rule did you use?

Since $\chi^2_{1,1-\alpha} = 3.841$



Since our χ^2 test statistic is greater than $\chi^2_{1,1-\alpha}$ at $\alpha = 0.05$, we $\boxed{\text{reject the } H_0}$.

Step 6. State your conclusion.

We conclude that Drug A and Drug B are not equally effective.

In the same study, if the focus is on the 100 matched pairs consisting of male patients, then the following results are obtained: for 52 matched pairs, both drugs are effective; for 35 matched pairs both drugs are ineffective; for 4 matched pairs drug A is effective whereas drug B is ineffective; and for 9 matched pairs drug B is effective, whereas drug A is ineffective.

(c) Draw the new matched pairs table.

		Treatment B	
		Effective	Not Effective
Treatment A	Effective	52	4
	Not Effective	9	35

100

(d) How many concordant pairs are there among the male matched pairs?

52

(e) How many discordant pairs are there among the male matched pairs?

$n_D = n_A + n_B = 4 + 9 = 13$

(f) In McNemar's Test, can you verify that $\frac{n_D}{4} \geq 5$ or $n_D \geq 20$ is satisfied? If this is not satisfied, what should we do?

$n_D = 13 < 20$ so we should use an Exact Test to find the p-value.

(g) Compute the p-value and state your conclusion.

Since $n_A = 4 < \frac{n_D}{2} = 6.5$, we use

$p = 2\sum_{k=0}^{n_A} \binom{n_D}{k}\left(\frac{1}{2}\right)^{n_D}$ to get the exact p-value

$$= 2\sum_{k=0}^{4} \binom{13}{k}\left(\frac{1}{2}\right)^{13}$$

$$= 2\left(\frac{1}{2}\right)^{13}\sum_{k=0}^{4}\binom{13}{k} = 2\left(\frac{1}{2}\right)^{13} 1093 \approx 0.267$$

$\sum_{k=0}^{4}\binom{13}{k}$ = sum(choose(13, 0:4)) in R

Conclusion: Since our p-value of 0.267 is greater than p=0.05, we fail to reject the null and conclude that for male patients, drug A and drug B are equally effective.